Additional material for the paper:

"Systematic Speedup in Convergence of Path Integrals",
A. Bogojević, A. Balaž and A. Belić

The paper and the Monte Carlo codes developed for it can be found at
http://scl.phy.bg.ac.yu/speedup/

Effective Action to level $p = 6$

The effective action at level $p = 6$ leads to amplitudes that differ from the continuum limit by a term of order $O(1/N^6)$ and is given by the following expression:

$$
\begin{aligned}
V_{p=6} &= V + \epsilon_N \frac{V''}{12} + \epsilon_N^2 \left[-\frac{V'^{\,2}}{24} + \frac{V^{(4)}}{240} \right] + \epsilon_N^3 \left[-\frac{V''^{\,2}}{360} - \frac{V'\,V^{(3)}}{120} + \frac{V^{(6)}}{6720} \right] + \\
&+ \epsilon_N^4 \left[\frac{V'^{\,2}\,V''}{240} - \frac{23\,{V^{(3)}}^2}{40320} - \frac{V''\,V^{(4)}}{1680} - \frac{V'\,V^{(5)}}{2240} + \frac{V^{(8)}}{241920} \right] + \\
&+ \epsilon_N^5 \left[\frac{V''^{\,3}}{5670} + \frac{29\,V'\,V''\,V^{(3)}}{20160} + \frac{V'^{\,2}\,V^{(4)}}{2240} - \frac{47\,{V^{(4)}}^2}{1209600} - \right. \\
&\qquad \left. - \frac{19\,V^{(3)}\,V^{(5)}}{241920} - \frac{V''\,V^{(6)}}{30240} - \frac{V'\,V^{(7)}}{60480} + \frac{V^{(10)}}{10644480} \right] \\
(g_1)_{p=6} &= \frac{V''}{24} + \epsilon_N \frac{V^{(4)}}{480} + \epsilon_N^2 \left[-\frac{V''^{\,2}}{1440} - \frac{V'\,V^{(3)}}{480} + \frac{V^{(6)}}{13440} \right] + \\
&+ \epsilon_N^3 \left[-\frac{{V^{(3)}}^2}{4032} - \frac{V''\,V^{(4)}}{5040} - \frac{V'\,V^{(5)}}{6720} + \frac{V^{(8)}}{483840} \right] + \\
&+ \epsilon_N^4 \left[\frac{V''^{\,3}}{60480} + \frac{V'\,V''\,V^{(3)}}{3360} + \frac{V'^{\,2}\,V^{(4)}}{13440} - \frac{13\,{V^{(4)}}^2}{806400} - \right. \\
&\qquad \left. - \frac{V^{(3)}\,V^{(5)}}{26880} - \frac{V''\,V^{(6)}}{80640} - \frac{V'\,V^{(7)}}{161280} + \frac{V^{(10)}}{21288960} \right] \\
(g_2)_{p=6} &= \frac{V^{(4)}}{1920} + \epsilon_N \frac{V^{(6)}}{53760} + \epsilon_N^2 \left[-\frac{{V^{(3)}}^2}{32256} - \frac{V''\,V^{(4)}}{40320} - \frac{V'\,V^{(5)}}{53760} + \frac{V^{(8)}}{1935360} \right] + \\
&+ \epsilon_N^3 \left[-\frac{{V^{(4)}}^2}{345600} - \frac{V^{(3)}\,V^{(5)}}{138240} - \frac{V''\,V^{(6)}}{483840} - \frac{V'\,V^{(7)}}{967680} + \frac{V^{(10)}}{85155840} \right] \\
(g_3)_{p=6} &= \frac{V^{(6)}}{322560} + \epsilon_N \frac{V^{(8)}}{11612160} + \\
&+ \epsilon_N^2 \left[-\frac{{V^{(4)}}^2}{4147200} - \frac{V^{(3)}\,V^{(5)}}{1658880} - \frac{V''\,V^{(6)}}{5806080} - \frac{V'\,V^{(7)}}{11612160} + \frac{V^{(10)}}{510935040} \right] \\
(g_4)_{p=6} &= \frac{V^{(8)}}{92897280} + \epsilon_N \frac{V^{(10)}}{4087480320} \\
(g_5)_{p=6} &= \frac{V^{(10)}}{40874803200} \,,
\end{aligned}
$$

where V is the potential of the starting action. All higher g_k's are zero at this level. The effective actions for lower levels can be obtained from the above expression by appropriate truncations (see paper).