Resistance in percolating quasi 1D and 2D networks of nanofibers

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(1) Motivation

Percolation is random process, and we can distinguish two types of the percolation problems: lattice percolation and continuum (or irregular lattice) percolation. It is widely accepted that lattice and continuum percolation belong to the same class in the sense that the latter possesses the same critical exponents as the former.

Stick percolation has not been extensively studied theoretically until now [1, 2]. Still it is an important representative of continuum percolation and interesting due to its relevance for systems consisting of conducting rodlike particles.

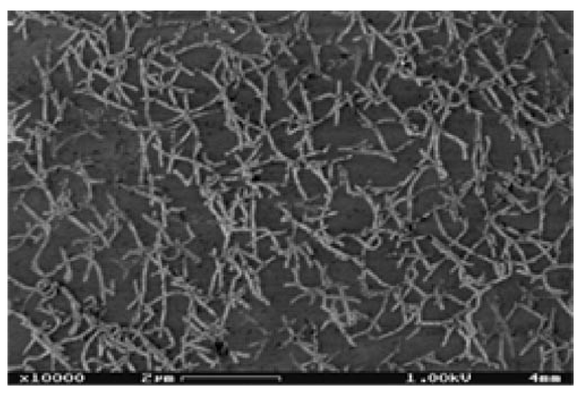
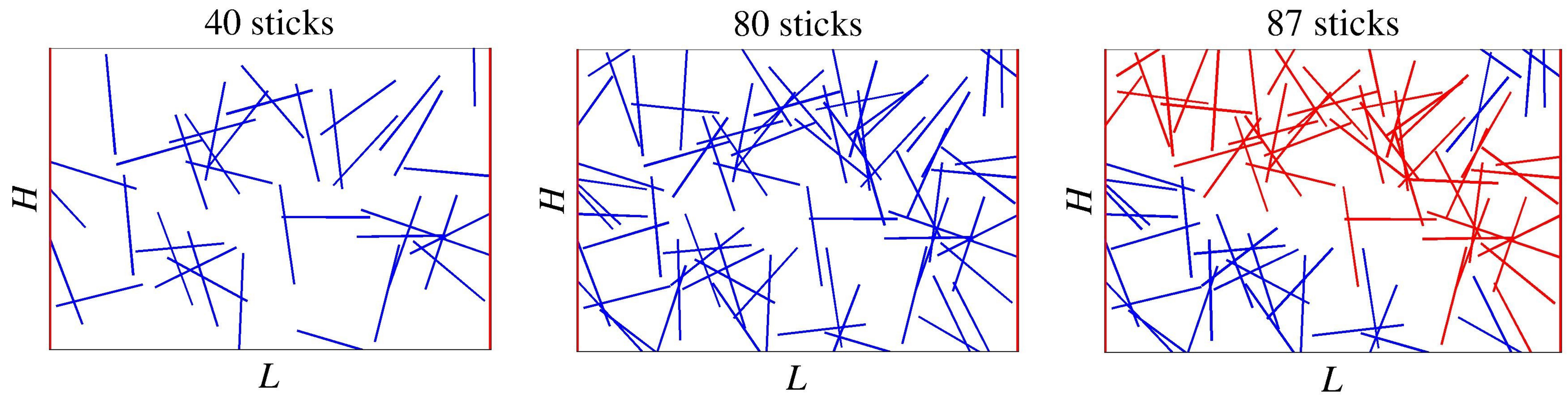


Illustration of carbon nanotube network taken from M.Y. Zavodchikova et. al., Nanotechnology 20 (2009) 085201.

(2) Monte Carlo simulations of rodlike particle percolation

In Monte Carlo simulations the widthless sticks with unity length are randomly placed between electrodes (left / right) with free boundary conditions (top / bottom). Two sticks lie in the same cluster if they intersect. System percolates if two opposite boundaries (left and right) are connected with the same cluster.



Important properties of the system:

1) Aspect ratio $r = L/H$.
2) Stick density *n*, i.e., the number of sticks per unit area $n = N r / L^2$.

(3) Results for probability distribution

(A) Conversion of number of sticks *N* into stick density *n*

Convolving the percolating probability $R_{N,L,r}$ for *N* sticks in a system with the Poisson distribution we obtain a percolating probability for any stick density *n* :

$$R_{n,L,r} = \sum_{N=0}^{\infty} \frac{(n L^2/r)^N e^{-nL^2/r}}{N!} R_{N,L,r}.$$

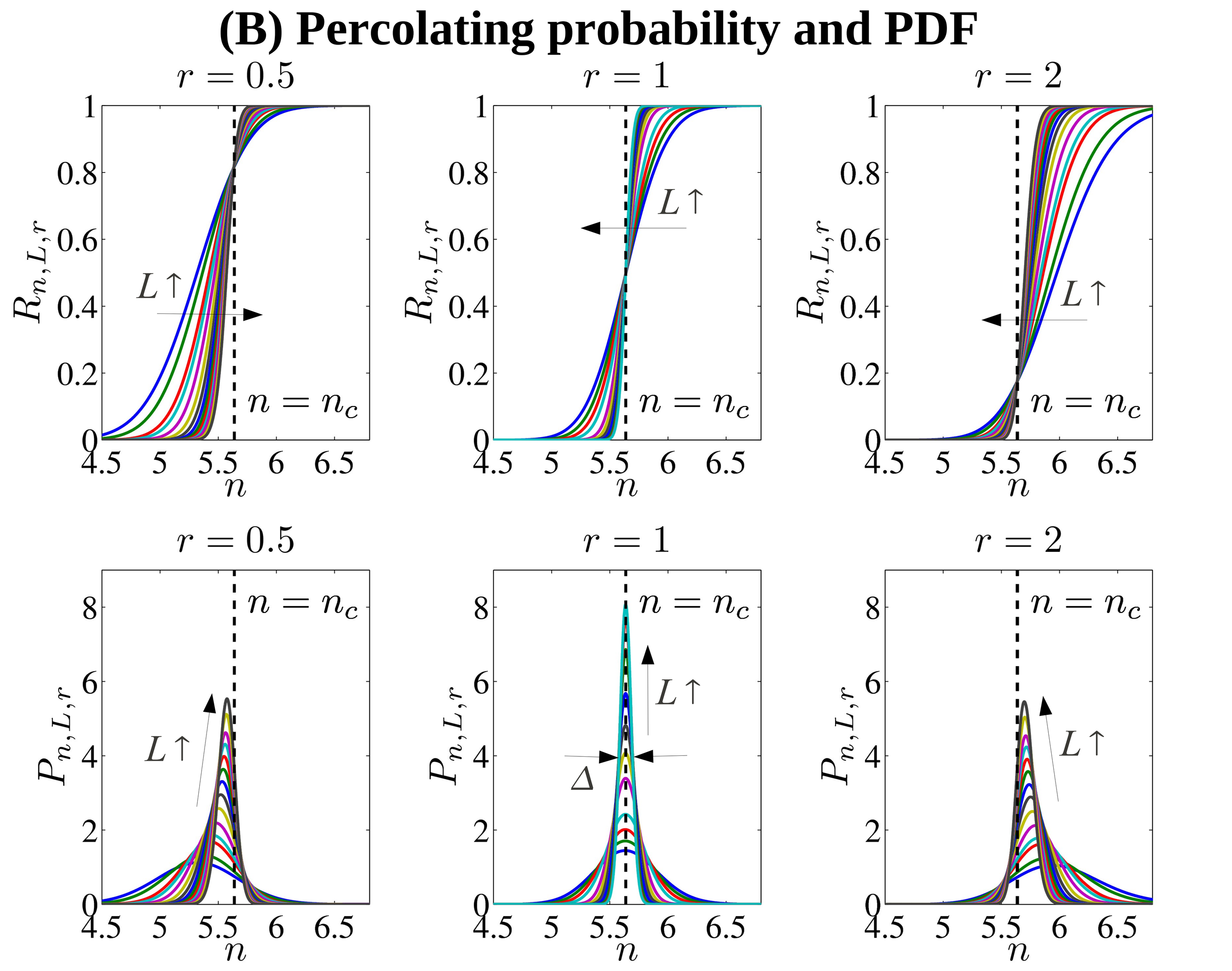
Probability density function (PDF):

$$P_{n,L,r} = \frac{dR_{n,L,r}}{dn}.$$

Recently has been shown that threshold for stick percolation should be [1]:

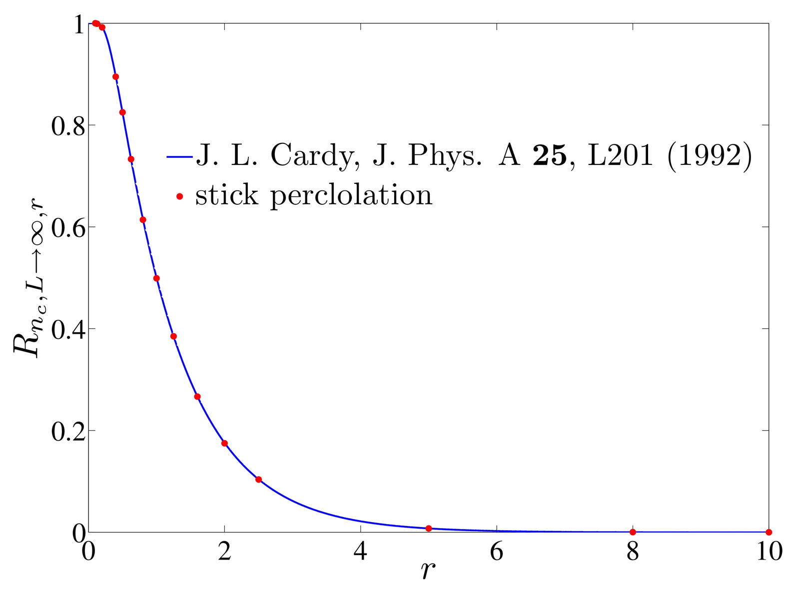
$$n_c = 5.63726 \pm 0.00002.$$

(B) Percolating probability and PDF



(C) Comparison of infinite system with analytical model for lattice percolation

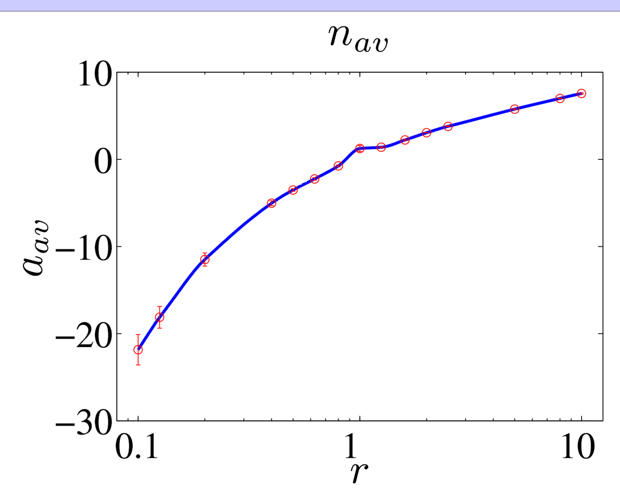
Percolating probability at percolation threshold shows excellent agreement with Cardy's prediction for lattice percolation.

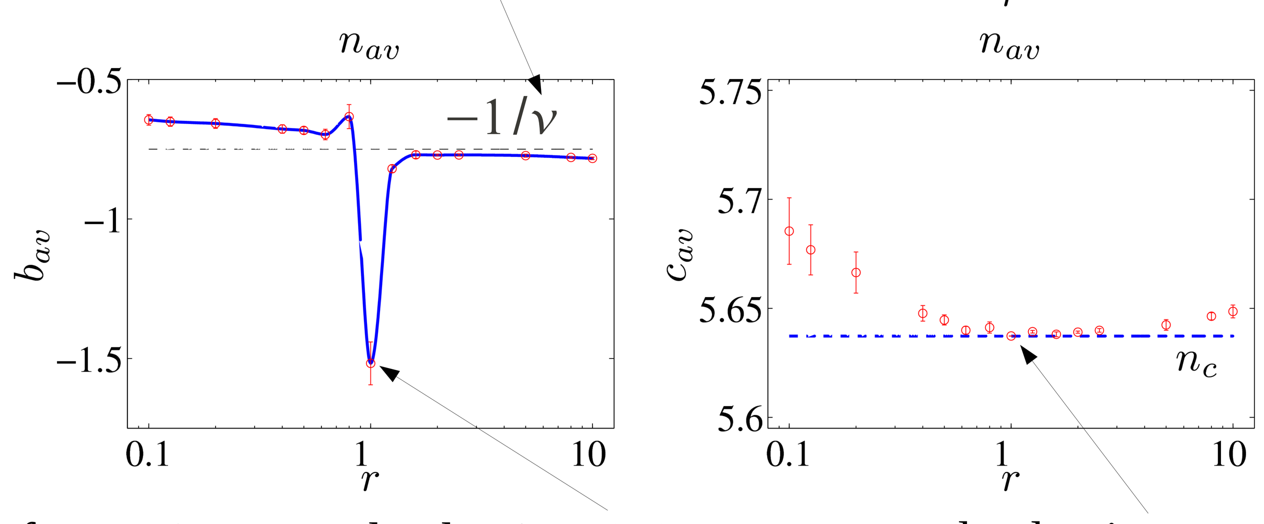


(4) Average stick density

$$n_{av} = \int_{n=0}^{\infty} n P_{n,L,r} dn$$
$$\approx a_{av} L^{b_{av}} + c_{av}$$

universal exponent for two dimensional systems: $\nu = 4/3$





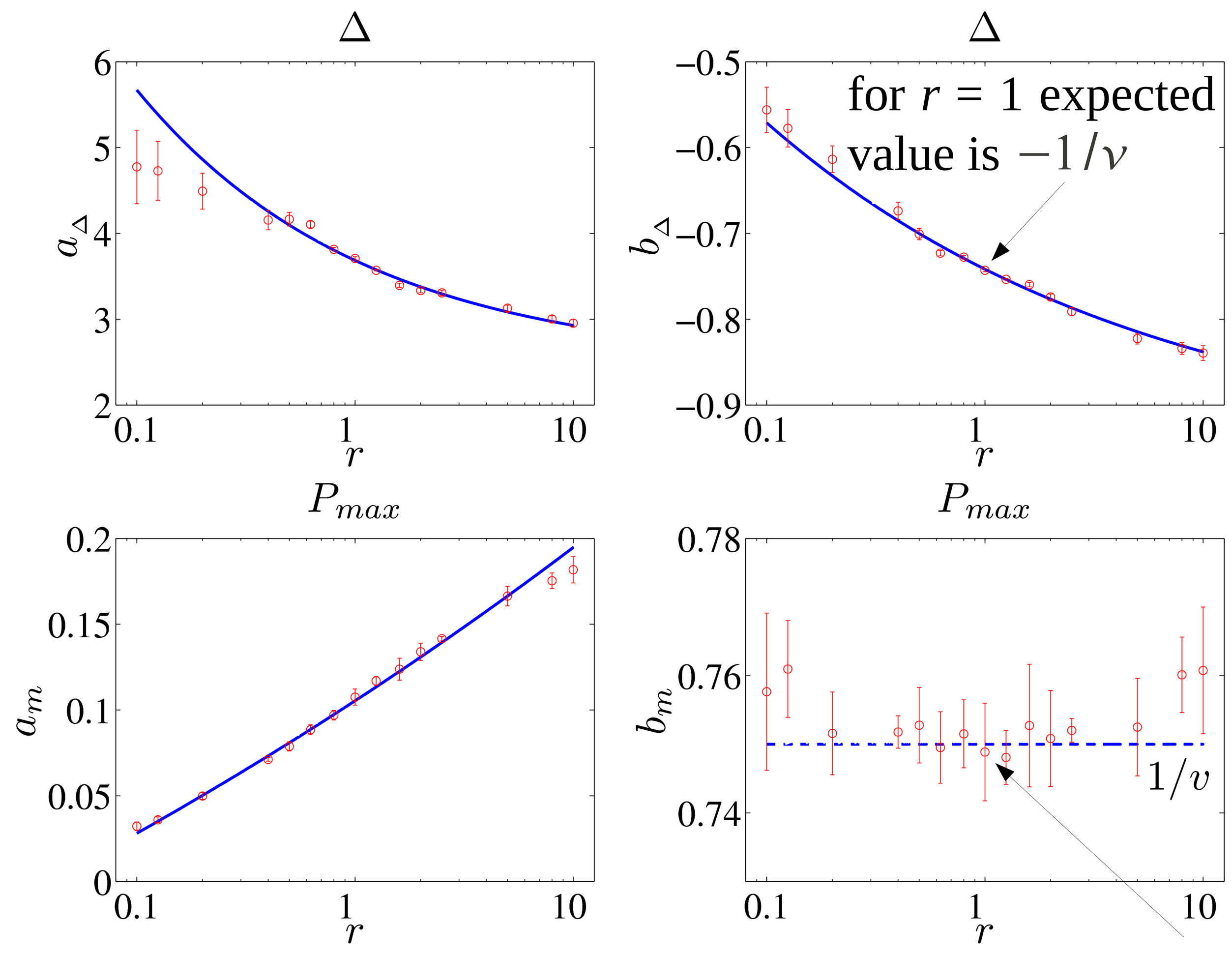
for *r* = 1 expected value is -1.65 　 expected value is n_c

(5) Standard deviation and maximum of PDF

$$\Delta^2 = \int_{n=0}^{\infty} (n - n_{av})^2 P_{n,L,r} dn$$
$$\approx a_\Delta L^{b_\Delta}$$

$$P_{max} = max(P_{n,L,r})$$
$$\approx a_m L^{b_m}$$

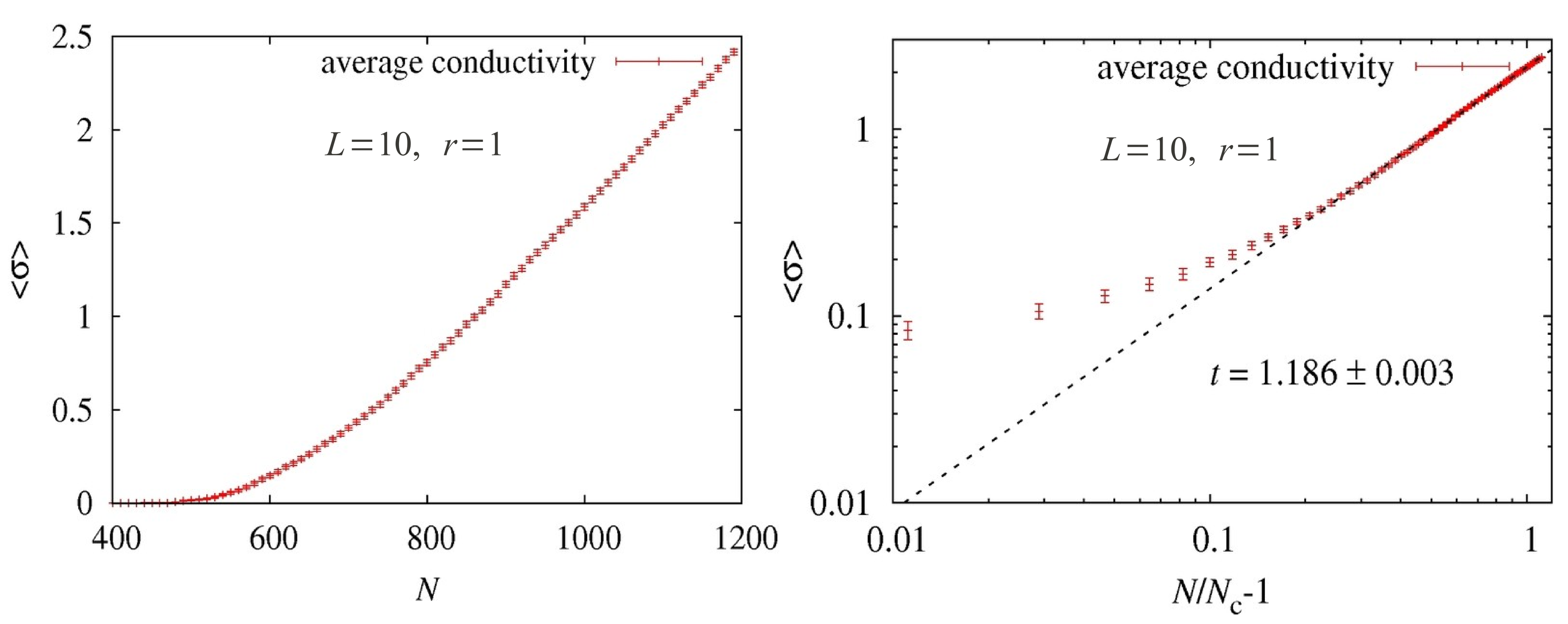
Understanding standard deviation and PDF is important for quality control in applications.



for *r* = 1 expected value is $1/\nu$

(6) Conductivity exponent in stick percolation

Average conductivity $\langle \sigma \rangle = (n - n_c)^t$, $L \gg \xi$.
$\xi \propto |n - n_c|^{-\nu}$ is the correlation length.



References:
[1] J. Li and S.-L. Zhang, Phys. Rev. E **79**, 155434 (2009)
[2] J. Li and S.-L. Zhang, Phys. Rev. E **79**, 021120 (2010)
[3] D. Stauffer and A. Aharony, *Introduction to Percolation Theory*, 2nd revised ed. Taylor and Francis, London, 2003

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