



## Bipartite networks

Jelena Grujić

Motivation

Movie recommendations

Model

Conclusion

# Modeling real networks through bipartite graphs

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# Outline

Bipartite  
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Motivation

Movie recom-  
mendations

Model

Conclusion

## 1 Motivation

## 2 Movie recommendations

## 3 Model

## 4 Conclusion



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**Bipartite  
networks**

Jelena Grujić

**Motivation**

Movie recom-  
mendations

Model

Conclusion

## 1 Motivation

## 2 Movie recommendations

## 3 Model

## 4 Conclusion



# Bipartite Networks

Bipartite networks

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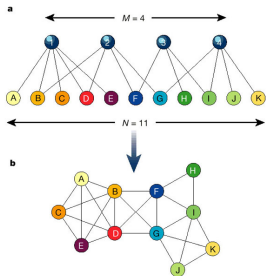
Motivation

Movie recommendations

Model

Conclusion

- Two types of nodes
- Links only between nodes of different type
- One-mode projections
- Why do we investigate them?





# Examples

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## Motivation

Movie recommendations

Model

Conclusion

- Protein-protein interaction: bait and prey
- Gene regulation: regulators and genes
- Scientific publication networks: scientist and papers
- Actors network: actors and movies
- Corporate directors: boards and directors
- Economy networks: banks and companies
- Language networks: words and sentences



# New input

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### Motivation

Movie recommendations

Model

Conclusion

- Two possible projected networks: top and bottom
- Trivial one-mode projection
- Many common nodes per pair
- Weighted one-mode projection
- Underlying bipartite structure



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**Bipartite  
networks**

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Motivation

**Movie recom-  
mendations**

Model

Conclusion

**1 Motivation**

**2 Movie recommendations**

**3 Model**

**4 Conclusion**



# E-Social Networks

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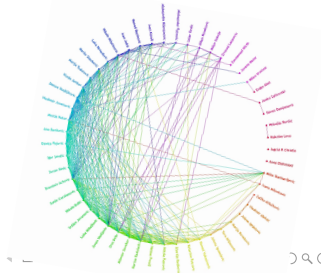
Motivation

Movie recommendations

Model

Conclusion

- Interactions between humans or groups over the internet
- E-mail, chats, forums, facebook,...
- Recommendations networks
- Goal 1: Get to know social structure
- Goal 2: Use that knowledge to develop recommendation system
- Here: Example of bipartite networks
- Movies recommendations







# The Internet Movie Database



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Motivation

Movie recom-  
mendations

Model

Conclusion

- On-line database about films, TV products, direct to video products and video games
- Launched 1990., “Movie rating system” and “Those Eyes”
- Commercial 1995., Amazon.com 1998.
- Today: Titles 1,039,447
- Movies released theatrically: 379,871
- Users: votes, comments, message boards etc.
- Recommendations lists
- USA movies, more than 10 votes, excluded TV, straight to video and video games, around 43,000 titles, 350,000 users



# Investigated networks

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Movie recommendations

Model

Conclusion

### ■ IMDb recommendations

- From recommendation lists on the site. User votes, genre, title, keywords, and, most importantly, user recommendations themselves

### ■ UD-BP - User Driven Bipartite networks

- Movie and user are linked if specific user left a comment on the web page of specific movie.

### ■ WOMP - Weighted One-mode projections

- Recommendation list - ten movies with highest value of *Cosine*.

$$\mu_i = (\dots, 1, \dots, 0, \dots, 0, \dots, 1, \dots)$$

$$Cosine_{ij} = \frac{\vec{\mu}_i \cdot \vec{\mu}_j}{|\mu_i| |\mu_j|}$$



# Examples - IMDb

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Motivation

Movie recom-  
mendations

Model

Conclusion

- **The Lord of the Rings: The Fellowship of the Ring (2001)**  
The Lord of the Rings: The Return of the King (2003) The Lord of the Rings: The Two Towers (2002) The Chronicles of Narnia: The Lion, the Witch and the Wardrobe (2005) Stardust (2007) Willow (1988) Indiana Jones and the Temple of Doom (1984) Beowulf (2007) Details Cinema Showtimes Star Wars: Episode III - Revenge of the Sith (2005) The Lord of the Rings (1978) The Return of the King (1980) (TV)
- **Donnie Darko (2001)**  
The Butterfly Effect (2004) The Living and the Dead (2006) The Basketball Diaries (1995) Disturbia (2007) Vana espuma (1998) Blue Velvet (1986) Requiem for a Dream (2000) Twin Peaks: Fire Walk with Me (1992) Trick or Treat (1986) Vanilla Sky (2001)



# Examples - WOMP

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Motivation

Movie recom-  
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Model

Conclusion

- **The Lord of the Rings: The Fellowship of the Ring (2001)**  
The Lord of the Rings: The Return of the King (2003) The Lord of the Rings: The Two Towers (2002) Ocean's Eleven (2001) Shrek (2001) Spider-Man (2002) Star Wars (1977) Star Wars: Episode I - The Phantom Menace (1999) Star Wars: Episode II - Attack of the Clones (2002) Star Wars: Episode V - The Empire Strikes Back (1980) Star Wars: Episode VI - Return of the Jedi (1983)
- **Donnie Darko(2001)** Insomnia (2002/I) One Hour Photo (2002) The Others (2001) Out of Time (2003/I) Panic Room (2002) Phone Booth (2002) The Ring (2002) Road to Perdition (2002) Secretary (2002) Vanilla Sky (2001)



# Degree distribution

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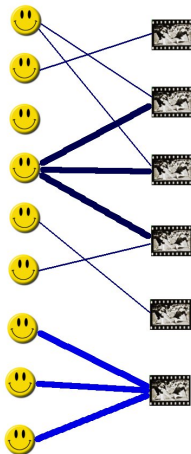
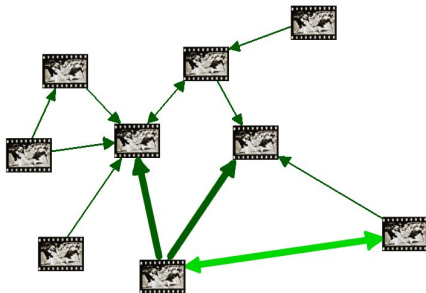
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Movie recommendations

Model

Conclusion

- Degree - number of neighbours
- Power law
- Per movie:  $k_{max} = 4,763$ ,  $\langle k \rangle = 27$
- Per user:  $k_{max} = 3,040$ ,  $\langle k \rangle = 3$





# Degree distributions monopartite networks

Bipartite networks

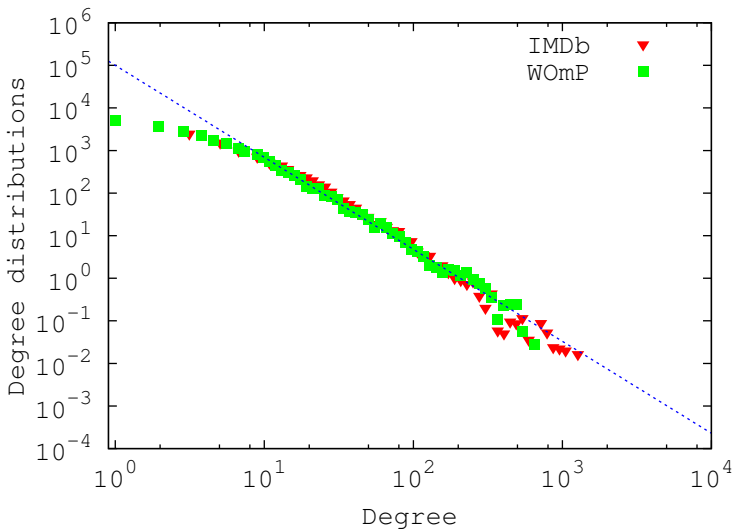
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Movie recommendations

Model

Conclusion





# Degree distributions bipartite networks

Bipartite networks

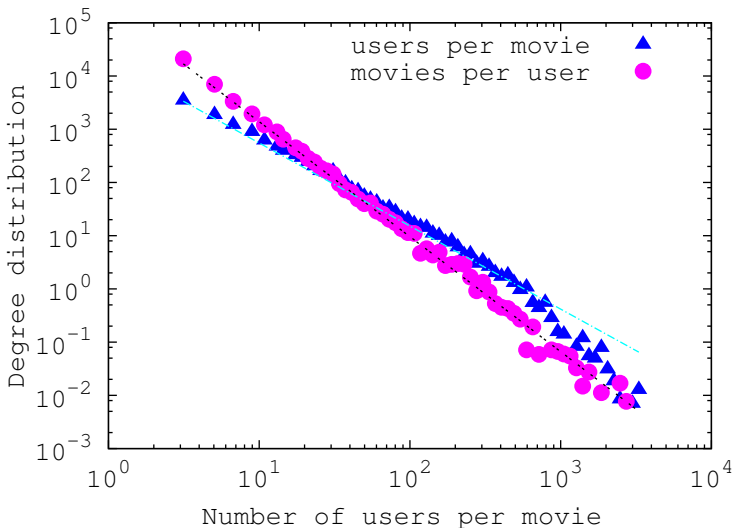
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Movie recommendations

Model

Conclusion





# Clustering coefficient

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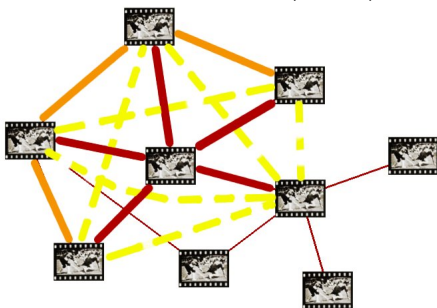
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Model

Conclusion

- Clustering coefficient - how connected are the neighbours
- Social networks - high clustering

$$c_i = \frac{2e_i}{k_i(k_i - 1)} = \frac{2 \cdot 3}{5 \cdot 4} = 0.3$$



$$C_{IMDb} = 0.19$$

$$C_{WOMP} = 0.54$$





# Distribution of Commons - bipartite clustering

Bipartite networks

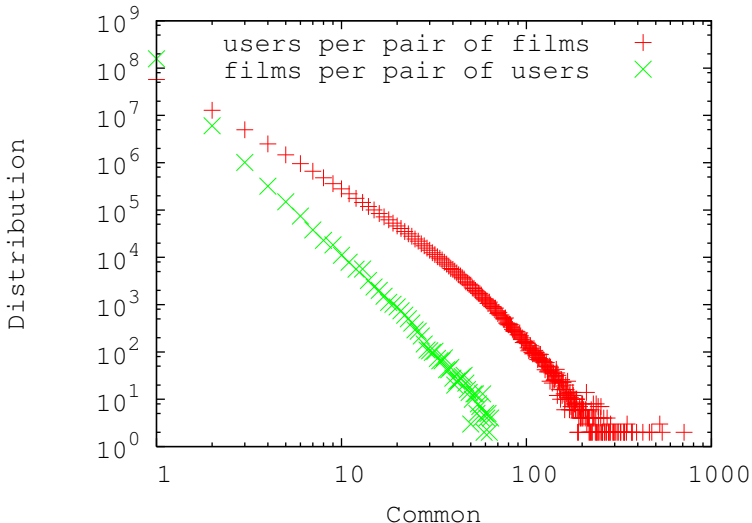
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Movie recommendations

Model

Conclusion





# Distribution of Cosines

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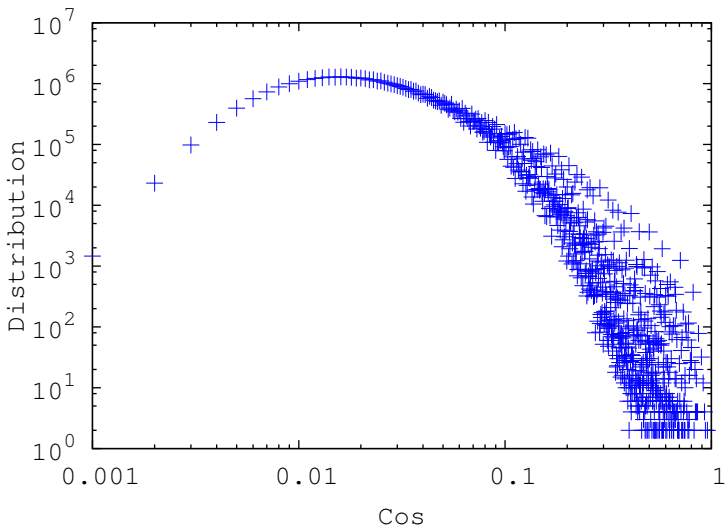
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Movie recom-  
mendations

Model

Conclusion





# Assortativity

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mendations

Model

Conclusion

- Average nearest neighbours degree

$$k_{nn,i} = \frac{1}{k_i} \sum_{j=1}^N a_{ij} k_j$$

- Assortative connect to similar - social, biological
- Disassortative connect to different - technological



# Assortativity monopartite networks

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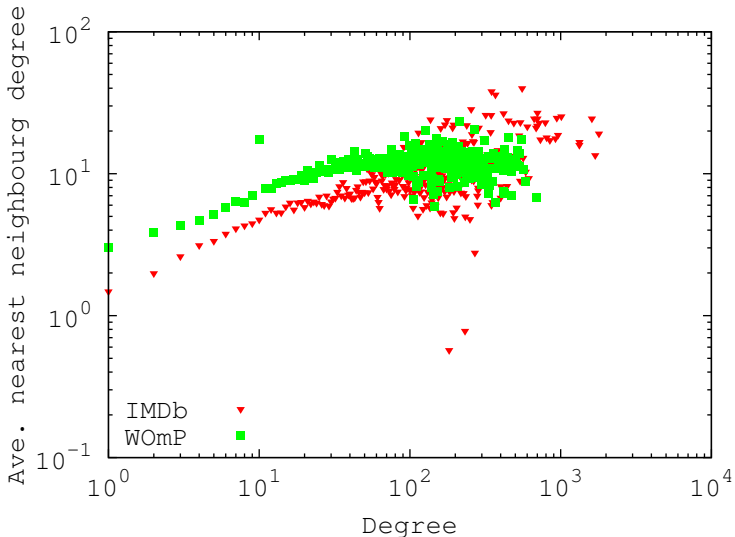
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Motivation

Movie recommendations

Model

Conclusion





# Assortativity bipartite networks

Bipartite networks

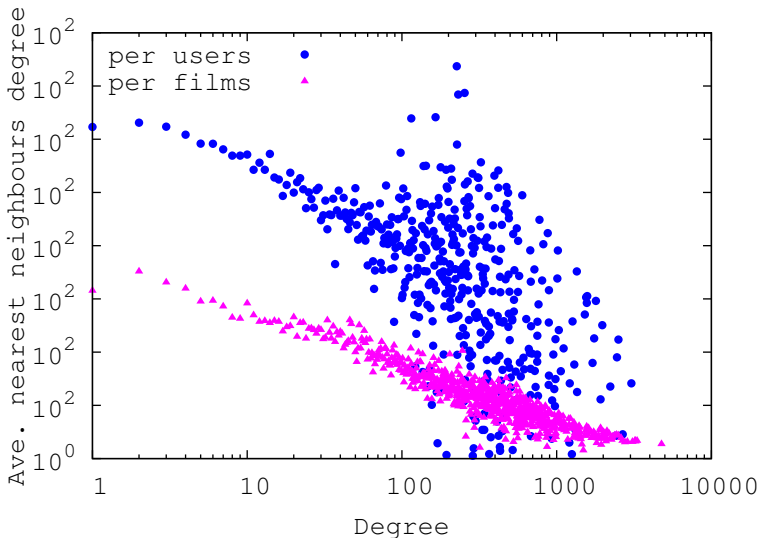
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Motivation

Movie recommendations

Model

Conclusion





# Outline

**Bipartite  
networks**

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Motivation

Movie recom-  
mendations

**Model**

Conclusion

**1** Motivation

**2** Movie recommendations

**3** Model

**4** Conclusion



# Model

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Motivation

Movie recommendations

Model

Conclusion

- Analytical and numerical approach
- Guillaume, Latapy, Informational Processing Letters (2004)
- Degree distribution, Clustering
- High number of commons
- Model: Preferential growth + Exchange of opinion

$$\frac{c_{ij} + 1}{\sum_{a \neq i} c_{ia} + 1}$$



# Degree distribution

Bipartite networks

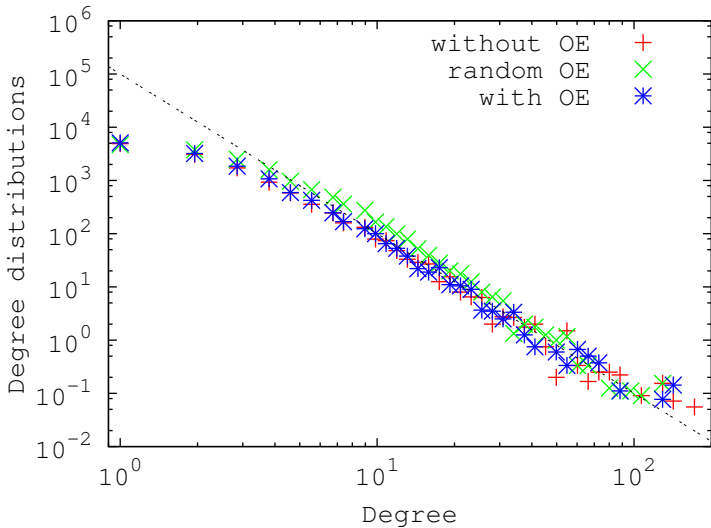
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Motivation

Movie recommendations

Model

Conclusion







# Distribution of Commons

Bipartite networks

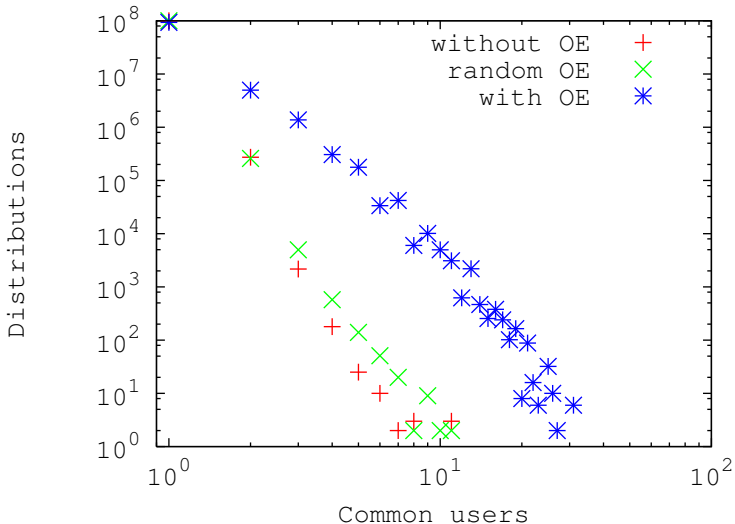
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Motivation

Movie recommendations

Model

Conclusion





# Outline

**Bipartite  
networks**

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Motivation

Movie recom-  
mendations

Model

**Conclusion**

**1** Motivation

**2** Movie recommendations

**3** Model

**4** Conclusion



# Conclusion

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Motivation

Movie recommendations

Model

Conclusion

- Two type of networks considered: monopartite and bipartite
- Monopartite: power law degree distribution, high clustering, assortative mixing, similar properties
- Bipartite: power law degreedistributions, high number of commons, disassortative mixing
- Scale free degree distribution  $\longrightarrow$  Scale free degree distribution
- Disassortative mixing  $\longrightarrow$  Assortative mixing
- Model with exchange of opinion



# Future works

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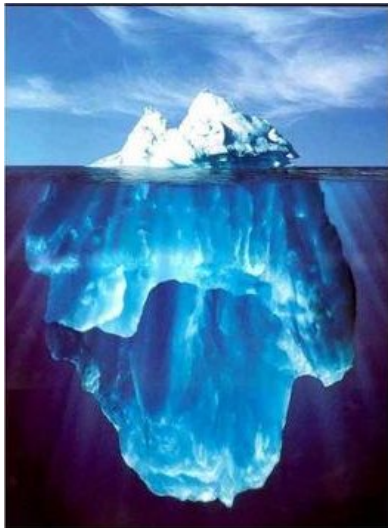
Motivation

Movie recom-  
mendations

Model

**Conclusion**

- Analytical solution of the model
- Fine tune model
- Assorativity model
- Modular structure



# Acknowledgments

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Motivation

Movie recom-  
mendations

Model

Conclusion



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